Wavelet transform based damage detection in a plate structure

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Problem statement

Solution

nondestructive structural health monitoring methods

taken from www.ptclwg.com
taken from www.rbengineering.com
Problem statement

- We propose a method for damage identification based on transform of structural vibration modes

\[ \text{Continuous Wavelet Transform (CWT) in space domain} \]
aluminium plate
(1000 x 1000 x 5 mm)

**ANSYS model** – 8-node shear deformable shell elements.

- 100 x 100 equal length elements;
- $E = 69$ GPa, $\nu = 0.31$, $\rho = 2708$ kg/m$^3$;
- **Damage** - reduction of flexural stiffness of the selected elements (decrease in element thickness).
Numerical simulation

**Input**
- Plate numerical data.

**Modal analysis**
- 12 mode shapes.

**Damage detection algorithm**
- Sharp large amplitude peaks in damage index profiles – zone of damage.
Algorithm

Generalized 2D-CWT for isotropic wavelet

1. \( W_{s,a,b} = \frac{1}{\sqrt{s \cdot s}} \iint f(x, y) \cdot \psi^\ast \left( \frac{x-a}{s}, \frac{y-b}{s} \right) dx dy = \iint f(x, y) \cdot \psi^\ast_{s,a,b}(x, y) dx dy \)

Damage index 2D-mode shape data

2. \( DI_{i,j}^{n_{2DCWT}} = W_{i,j}^{n} = \iint w_{i,j}^{n} \cdot \psi^\ast_{s,a,b}(x, y) dx dy \)

3. \( DI_{i,j} = \frac{1}{N} \sum_{n=1}^{N} \frac{DI_{i,j}^{n}}{DI_{i,j,\max}^{n}} \)

Standardized damage index

4. \( SDI_{i,j} = \frac{DI_{i,j} - \mu_{DI}}{\sigma_{DI}} \)

Damage estimate reliability

5. \( \text{DER} = 100\% \cdot \frac{(J \cdot I)^{-1} \sum_{j=1}^{J} \sum_{i=1}^{I} SDI_{i,j}}{(M \cdot N)^{-1} \sum_{j=1}^{M} \sum_{i=1}^{N} SDI_{i,j}} \)
Sensor density

• It is often not possible to equip a structure with dense sensor grid.

• Input mode shape data was divided by integer numbers $p = 1:8$.

\[
\text{Sensor density} = \text{size}(w_{ij}) \cdot p^{-1}
\]
• RESULTS
Best wavelet

Pet Hat wavelet (in frequency domain)

\[ \psi(\omega_x, \omega_y) = \begin{cases} 
\cos^2 \left( \frac{\pi \ln(\omega_x + i \omega_y)}{2 \ln(2)} \right) & \rightarrow 0.5 < |\omega_x + i \omega_y| < 2 \\
0 & \text{otherwise}
\end{cases} \]

expression adopted from Wavelet Toolbox, Matlab™
1st scale yields 98.16% DER
Sensor density increases
$p = 1$

101 × 101 points

SDI

mode shape sum
\[ p = 3 \]

![Graph showing mode shape sum for 34x34 points SDI](image)
$$p = 5$$

21 × 21 points

SDI

mode shape sum
\[ p = 8 \]

mode shape sum

13\times13 points

SDI
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Conclusions

• Spatial 2D -CWT is a reliable tool for damage localization in plate structures.

• Overall, 20 different wavelet functions were tested at scales 1:32.

• Best wavelet – Pet Hat wavelet (isotropic) with DER = 98.16 % at 1st scale.
Conclusions

- SDI profiles reveal the location of damage at different sensor densities.

- No clear trend of DER vs sensor density.

- DER vs Scale analysis is necessary to determine the appropriate wavelet scale.
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Thank You for your attention!